A connectionnist Approach to the Hexagonal Mesh Generation

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Abstract

We present a connectionnist approach for the constrained optimization problem of adaptive meshing. It consists in covering an amount of resources, distributed on a spatial discretized map, with a grid of quadrilateral or hexagonal meshes in such a way they all homogeneously divide it. In this paper, we only consider hexagonal meshes for representing the receptive field of agents acting on a spatial environment. The solution that we propose is based on the Kohonen Self-Organizing Maps learning algorithm because of its ability to generate a topological map according to a probability distribution. We show that the connectionnist model leads to a very simple and straightforward solution for hexagonal mesh generation. Results obtained by simulation show the algorithm ability to find solutions for a wide range of density map distributions.

Key words: optimization problem, grid generation, self-organizing map, neural network.

AMS subject classifications: 49Q10, 65M50, 82C32.

1 Introduction

The problems encountered in the field of resource optimization deal with large amounts of data and large combinatorial costs. They need the development of efficient and appropriate techniques that generate good solutions in a reduced amount of time. This is the case of the adaptive mesh generation problem that we present in this paper as a generic problem of spatial resource optimization. It consists in covering an underlying discretized resource map with meshes representing the spatial receptive field of agents, according to its resource distribution. The problem then consists in designing a grid of triangular, quadrilateral or hexagonal meshes in such a way that the resource is homogeneously shared out between meshes. Adaptive mesh generation can be used in many to improve the precision of simulation-based computations by optimizing the choice of points to be considered in the simulation, relatively to the underlying resource. Numerous industrial applications need for an optimization of calculation points in order to minimize the computation time or to maximize the resulting precision (fluid mechanic applications, interference calculation for frequency assignment...). Most of the time, the initial data subject to the computation is situated like in geographic coordinates and altitudes for meteorological measurements, and it can be regularly distributed or not. While mesh generation is a large area of research [2, 3], few approaches exist on mesh generation according to a density distribution. For example [1, 6] deal with this subject but they don't deal with more complex meshes such as hexagonal ones. This paper presents how the Kohonen [4, 5] self-organizing map algorithm is suitable for hexagonal mesh generation. It is organized as follows. Section 2 introduces and specifies the mesh generatin problem. Section 3 presents the connectionnist self-organizing map solution to the mesh generation problem. Section 4 gives three examples to illustrate the mesh generation solution. Section 5 is devoted to a summary and to prospects for further research.

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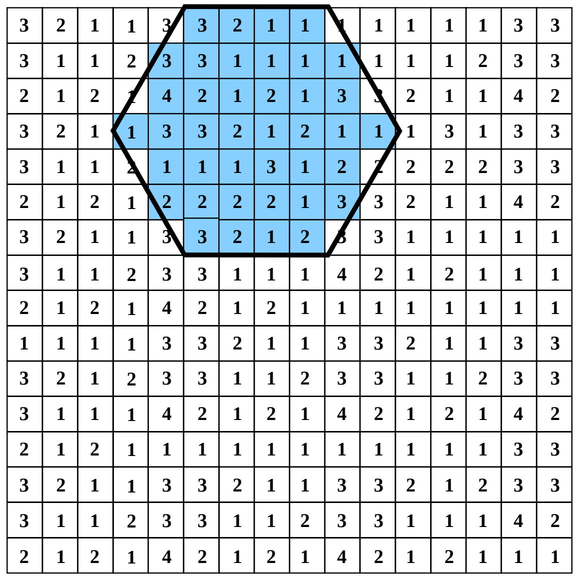

Figure 1: Hexagonal mesh generation: data and principle.

2 Hexagonal mesh generation

The goal in hexagonal mesh generation is to homogeneously divide a given numerical resource, located on a 2-dimensionnal discretized space, between meshes. Furthermore, the adapted meshes have to respect geometric constraints of shape. The process of adaptive meshing (AM) is then seen as a constrained optimization problem that enable one to position meshes in such a way as to optimally divide a resource between them. When meshing a surface, we need to consider both resources on the surface to be meshed, the size of the adapted meshing (AM) and geometrical constraints on adapted meshes. We thus present the notation we adopt in this paper and the formalization of these aspects. The resource density distribution is discretized on a fine-grained map of size $m \times n$. For each cell c_{ij} of the map, the weight is the quantity of resource it represents, and it is noted w_{ij}, with $i \in [1, m]$ and $j \in [1, n]$. This is illustrated by Figure 1. The target adaptive meshing is the set of hexagonal meshes M_{kl}, with $k \in [1, p]$, $l \in [1, q]$, $p << m$, $q << n$. The weight W_{kl} of a mesh M_{kl} is defined as the sum of the weight w_{ij} of the underlying cells c_{ij} covered by the hexagonal mesh M_{kl}. Figure 1 shows an example of an hexagonal mesh and the resource it covers.

Computing the homogeneity of resource distribution between the $p \times q$ meshes implies to determine for each mesh which underlying cells are covered by it through a pixel coloring algorithm. It is then possible to compute the distance from its actual weight W_{kl} to an ideal weight W. The ideal weight W of a mesh and the global cost CAM to minimize are defined as in equation (1).

$$(1) \qquad W = \frac{\sum_{k,l} W_{kl}}{p \times q}, C_{AM} = \frac{\sqrt{\sum_{k,l} (W_{kl} - W)^2}}{p \times q}$$

The criteria used to characterize the geometric constraints on a hexagon are based on convexity and regularity of the contour shape.

3 The connectionnist self-organizing map solution

The Kohonen [4,5] self-organizing map is an unsupervised learning network that preserves topological information about the input space. It can be considered as an undirect graph $G = (N, E)$, where a weight vector $w_n = (x, y) \in \mathcal{R}^2$ (i.e. the usual two dimensional euclidean space) is associated with every vertex or neuron $n \in N$. On the set of neurons of G we have the canonical metric d_G ($d_G(n, n') = 1$ iff they are connected by an edge) and the usual euclidean distance on $\mathcal{R}^2 : d(n, n^{'})$ as defined in equation (2). The receptive field Fn of a neurone $n \in N$ is given by the definition of a Voronoï region in equation (3).

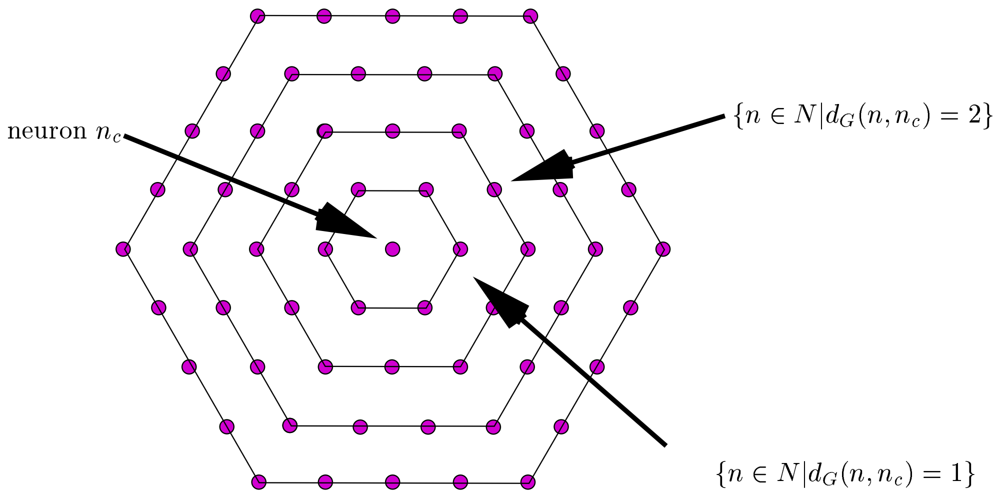

Figure 2: Hexagonal topology of the Kohonen map: each neuron is connected to its six adjacent neighbours.

(2) $$d(n, n^{'}) = \sqrt{(x - x^{'})^2 + (y - y^{'})^2}.$$

(3) $$F_n = \{w \in \mathcal{R}^2 | \forall n^{'} \in N, n^{'} \neq n, d(w, w_n) < d(w, w_{n^{'}})\}$$

The training procedure is applied as follows. At every training step a vector $x \in \mathcal{R}^2$ is presented according to the probability of density distribution. The neurone n_c is then chosen, for which $x \in F_{n_c}$ and the weight update rule is computed (in discrete time notation) for every neuron n in a neighborhood set N_c around cell n_c, according to equation (4).

(4) $$W_n(t + 1) = w_n(t) + \alpha(t).h_t(d_G(n_c, n)).(x(t) - w_n(t))$$

The update rule depends on the activation profile h_t and on the learning rate $\alpha(t)$. The activation profile h_t determines how strongly the weights of a neuron are corrected depending on its distance $d_G(n_c, n)$ from the maximally activated neuron n_c. This corresponds to biological lateral interaction and it has the shape of a "bell curve". $\alpha(t)$ and h_t are related to similar gains used in the stochastic approximation processes and as in these methods, $\alpha(t)$ and h_t should decrease with time.

The net topology defined by G is in our approach an hexagonal topology as shown in Figure 2. Generation of hexagonal meshes results from considering neurons as vertices and observing only the ones that constitute hexagonal meshes, i.e. an hexagon is defined by the set of neurons n at distance $d_G(n, n_c) = 1$ from a neuron n_c. Figure 3 illustrates the hexagonal meshes and the associated neural topology.

4 Simulation results

We now present three examples of optimization results. The nodes of the initial meshing are randomly chosen, at the exception of the ones on the outer edge of the resource area to be covered as shown in Figure 4. The nodes on the contour thus only translate along horizontal and vertical axes. The parameters of the algorithm learning rate and activation profile are the same for all the three examples. We have taken the learning rate $\alpha(t) = 0.9(1 - t/t_{max})$, where t_{max} is the total number of input presentations. The activation profile ht is given by equation (5) and the radius of the adjustment zone σ_t , which is a decreasing function of time, is given in equation (6).

(5) $$ht = exp(-d_G(n, n_c)^2 / \sigma_t^2)$$

(6) $$\sigma_t = \sigma_i(\sigma_f / .\sigma_i)^{t/t_{max}}$$

Figures 5-6-7 presents optimization results. Example Figure 5 deals with a 250x250 density map and a 15x15 hexagonal grid, Figure 6 deals with a 15x15 hexagonal grid on a 300x300 density map, and Figure 7 shows a 31x31

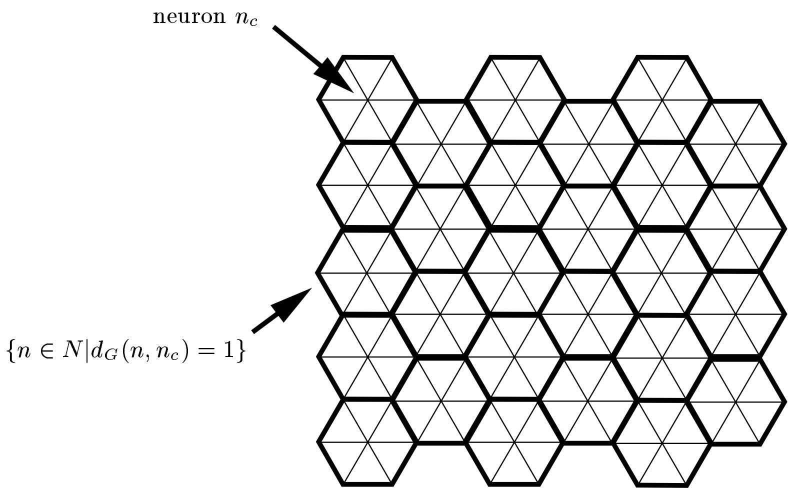

Figure 3: Hexagonal meshes and the underlying neural topology.

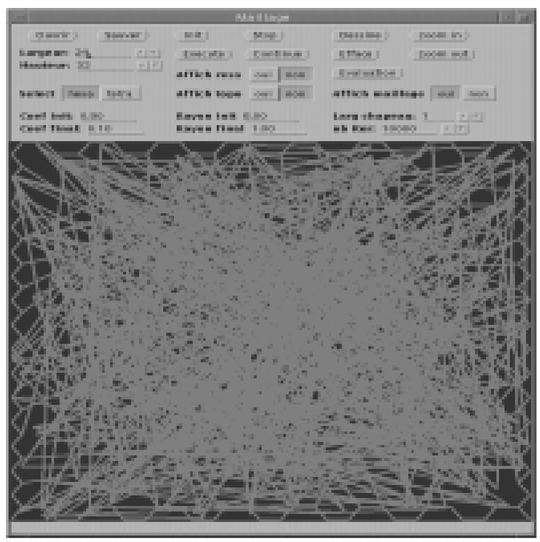

Figure 4: Initial random configuration of the meshing.

hexagonal grid on a 800x800 density map. We take $\sigma_i = 6$, $\sigma_f = 1$ and $t_{max} = 10000$ except for the third example where $t_{max} = 100000$. We can see that geometrical constraints on mesh convexity are respected and that the mesh surface depends on the local density intensity visible on the images. It lets us consider the ability for the algorithm to find good solution for a wide range of probability density distributions.

5 Conclusion

We have shown in this paper that Kohonen topological maps lead to a very simple and straightforward solution for the problem of hexagonal meshing. The geometrical constraints are naturally respected because of the hexagonal topology used and optimization of the adapted meshing is obtained by the algorithm ability to preserve the density distribution. As a target of further research, precise evaluations on concrete problems of resource distribution is envisaged. The quality of the density approximation given by the algorithm could then be specifically validated on real world cases.

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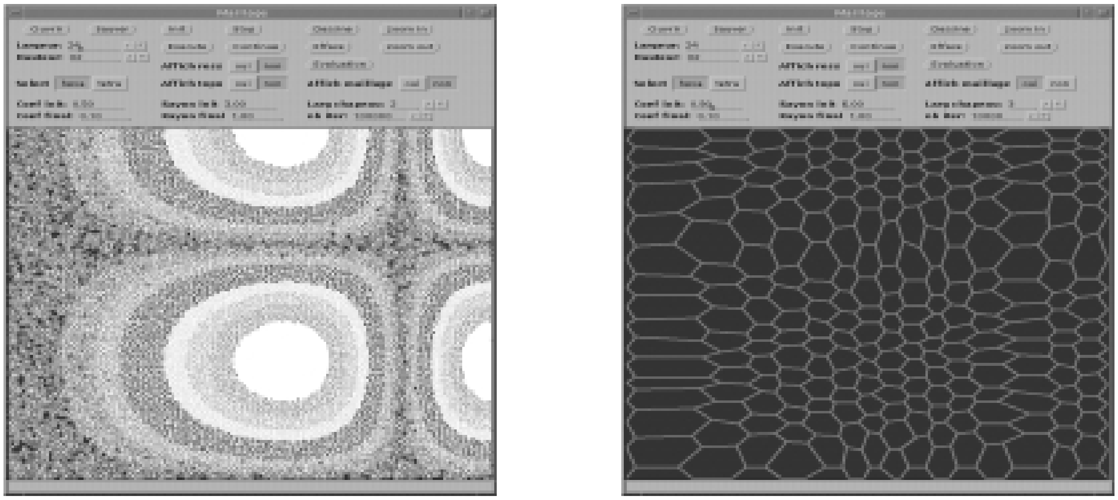

Figure 5: A 15x15 hexagonal grid on a 250x250 density distribution.

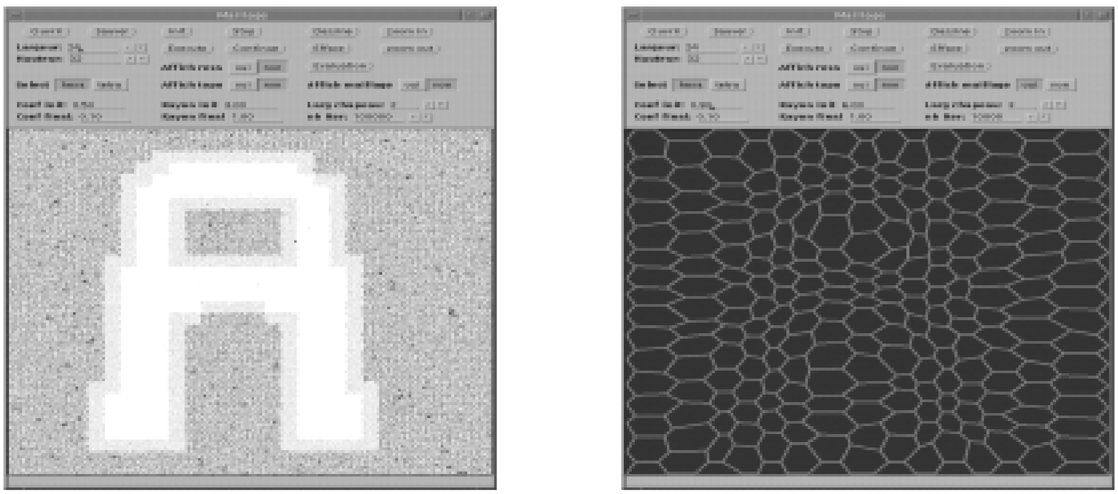

Figure 6: A 15x15 hexagonal grid on a 300x300 density distribution.

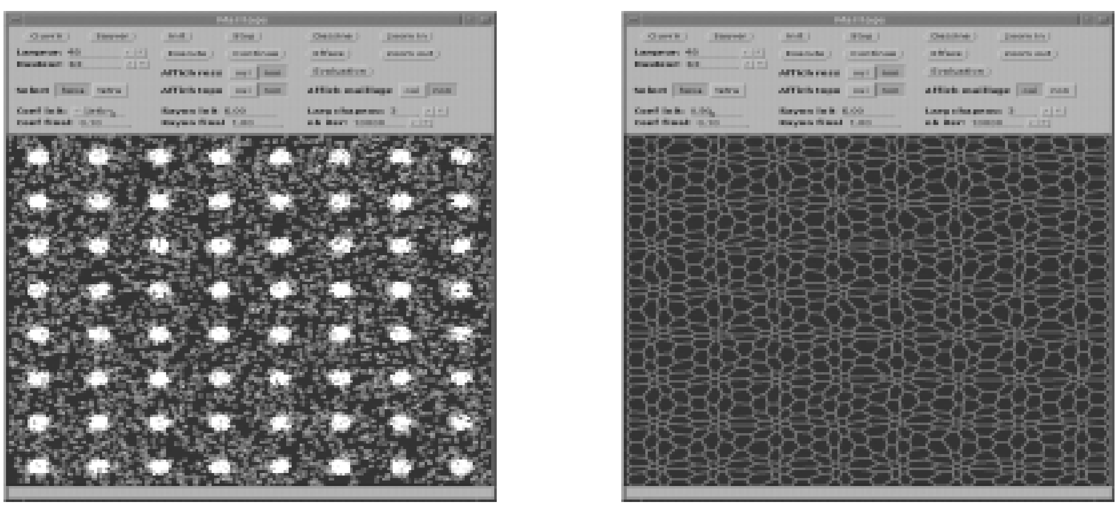

Figure 7: A 31x31 hexagonal grid on a 800x800 density distribution.