A Tabu search and Ant colony system Approach for the Capacitated Location-Routing Problem

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Abstract

In this article we study The Capacitated Location Routing Problem (CLRP) which is defined as a combination of two problems: the Facility Location Problem (FLP) and the Vehicle Routing problem (VRP). The CLRP is not just a purely academic construct; it has many applications in the practice. We propose a hybrid approach based on a Tabu search algorithm combined with an Improved Ant Colony System to solve the CLRP. The experimental results show the efficiency of our approach in comparison with the other methods found in the literature.

1. Introduction

The importance of studying the Capacitated Location Routing Problem (CLRP) is double: in one hand, it is very well-known problem in academic and theoretical research. In the other hand, it has many applications in the practice. The CLRP can be defined as a combination of two difficult problems: the Facility Location Problem (FLP) and the Vehicle Routing Problem (VRP). The CLRP considers a set of potential capacitated distribution centers (DC) and a set of ordered customers. The problem is also constrained with capacities on the vehicles. Each vehicle of the homogeneous fleet is assumed to visit a set of customers and each customer being visited once by one vehicle. The objective is to minimize the routing and the location costs.

Due to the complexity of the CLRP composed of two NP-hard problems: facility location and vehicle routing problem, many works are proposed to solve it using especially heuristic methods. Laporte, Nobert and Taillefer [8] review early work on location routing problems; they summarize the different types of formulations, solution algorithms and computational results of work published prior to 1988. More recently, a hierarchical taxonomy is developed and classification scheme is used to review the existing location routing literature [10]. The classification is based on the problem characteristics and solution methodology. Perl and Daskin [12] introduced the concept of iterating between location and routing phases. The location phase is formulated as an integer linear programming (ILP) and solved to optimality using implicit enumeration. The routing phase uses a savings-type heuristic generalised for multiple depots. Srivastava [13] also suggests three heuristics for the LRP, and explores the effects of several environmental factors on the algorithm performance. Tuzun and Burke [14] introduce a novel two-phase architecture that integrates both the location and routing decisions of the LRP. The two-phase approach coordinates two tabu search (TS) mechanisms - one seeking a good facility configuration, the other a good routing that corresponds to this configuration - in a computationally efficient algorithm. In this work, the capacity constraints are only considered on the vehicles. In our study we assume the capacity constraints on both depots and vehicles as in [1] where a cluster analysis based on sequential heuristic that uses simple procedures was presented. Moreover, four grouping techniques (hierarchical and non hierarchical) and six proximity measures were used to obtain several versions of the heuristic.

In this paper we present coordination of a Tabu Search (TS) and a hybrid Ant Colony system (ACS) [3] to the CLRP. The first seeks the good facility configuration, and the second determines a good routing that corresponds to this configuration. The remainder of this paper is organized as follows. In the section 2 the problem under investigated is formulated. Section 3 describes the approach proposed. Section 4 shows the computational results. Section 5 concludes the work.

2. Problem Formulation

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CLRP can be represented by a graph $Gr = (S, A)$, where $S = G \cup H$ is a set o vertices and A is a set of arcs. G denotes the set of indices of potential distribution centers and H the set of indices for the clients. For each $i \in G$, let f_i be the cost for opening the distribution center i. For each $j \in H$, q_j denotes the demand of client j. An assignment cost (routing cost) d_{ij} is associated to each arc $(i, j) \in A$. Figure 1 gives a graphical representation of the CLRP.

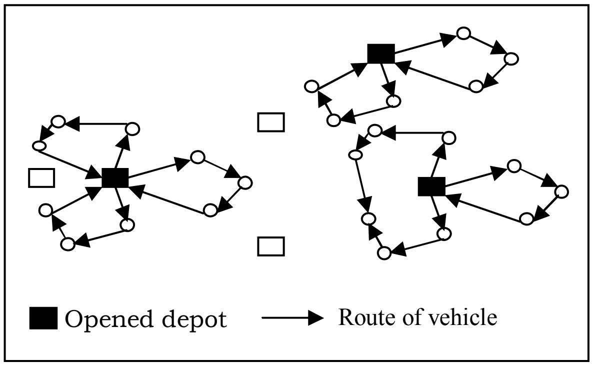

Figure 1 Example of CLRP solution

The objective is to find a set of distribution centers to be opened and a set of routes to service the clients from the open distribution centers in such a way that the opening costs plus the routing costs are minimized. We assume that 1) the capacity of each potential center distribution $i \in G$ is known; 2) the fleet of vehicles is homogeneous; 3) the routes start and end at the same facility; 4) each client is serviced by one and only one vehicle.

3. A Tabu search and Ant colony system algorithm for the CLRP

In our approach, the CLRP is divided into two phases: facility location phase and routing phase. We propose a meta-heuristic approach based on the tabu search and a hybrid ant colony system to solve the combined location routing problem. The tabu search (TS) is used to find a good configuration of distribution centers (DC), and the ant colony system (ACS) is used to find the good routing corresponding to this configuration. The two phases will be tackled repeatedly for a number of iterations. This two phase based approach offers a simple and a natural representation of the CLRP. The two meta-heuristics are coordinated so that an efficient exploration of the solution space is performed.

3.1. Initialization

A randomly selected distribution center is open, and all of the other candidate distributions centers closed. In this case, all the clients are assigned to the facility opened. And then we go to the location phase.

3.2. Location phase

The location phase tries to find a good configuration of distribution centers that allow reducing the cost of the last configuration. For the neighborhood system, in this phase, we apply two different types of moves: swap moves and add moves.

3.2.1. Swap Moves. These types of moves open one facility currently closed and close one of the facilities simultaneously. The swap moves explore the neighborhood to keep the number of open facilities in the solution constant, and search for a good configuration for a certain number of facilities, the location phase searches for the best swap move to perform. In order to select the best swap move, it is necessary to evaluate the cost of a swap move. The difference in the fixed cost when we open one facility and close another is straightforward (fixed cost of the facility to open - fixed cost of the facility to close). However, the difference in the routing cost is difficult to estimate. To do this, we take a simplistic approach, and assume that each customer is assigned to the closest open facility without violating the capacity constraint of facility. The difference in routing cost is then estimated using the difference in the direct distance between the customer and the facility according to the new and old assignments. The swap move evaluation is the sum of this routing cost estimate and the difference in the fixed cost. The swap move which yields the smallest evaluation is then performed, and both the move and its reverse are declared tabu for a number of iterations. After the swap move is performed, the search resumes to the routing phase to update the routing according to the swap move.

3.2.2. Add Moves. Having explored the configurations with the current number of facilities using the Swap moves, the search mechanism then increases the number of facilities by applying an add move. An add move opens one of the currently closed facilities, and therefore increases the number of facilities by one. Here, the routing cost is again estimated using the difference in direct distances for the customer assignments before and after the add move. Since opening a facility can only improve the routing cost estimate, this cost is always negative. The fixed cost of the facility to be opened is then added to the routing estimate in order to calculate the overall cost estimate. As in the swap move, the search again returns to the routing phase in order to update the routing after the add move. After one add move, the search continues with a series of swap moves until the termination criterion is satisfied.

3.3. Routing phase

After each swap or add move is performed, the routing phase is started from the best routing found for the previous facility configuration in order to modify the routing according to the current facility configuration. First, the customers are reassigned to the closest open facility with respecting the capacity constraint of the facilities. We obtain a certain number of clusters. A cluster is composed of one facility and a set of clients which are affected to this facility. We note that, the sum of customers' demand of each cluster doesn't exceed to the capacity of the facility. For each cluster, an ACS is executed to calculate the routing cost. The total cost is equal to the sum of routing cost of each cluster and the cost of opened facilities. If the total cost is smaller than the current best, we update the value of the best to the new value found and we go to the location phase. The algorithm is repeated for certain number of iterations.

In this section we describe the algorithm based on the Ant Colony System which is applied in each cluster to evaluate the routing cost:

3.3.1. Construction of vehicle routes. Initially, m ants are positioned on n customers randomly and initial pheromone trail levels are applied to arcs. In order to solve the CVRP, artificial ants construct solutions by successively choosing a customer to visit, continuing until each customer has been visited. When constructing routes if all remaining choices would result in an infeasible solution due to vehicle capacity being exceeded then the depot is chosen and a new route is started. Ants choose the next city to visit using a combination of heuristic and pheromone information. During the construction of a route the ant modifies the amount of pheromone on the chosen arc by applying a local updating rule. Once all ants have constructed their tours then the amount of pheromone on arcs belonging to the best solution, as well as the global best solution, are updated according to the global updating rule.

The probabilistic rule used to construct routes is as follows. Ant k positioned on node i chooses the next customer j to visit with probability $p_k(i, j)$ given in Equation (1).

$$j = \begin{cases} \operatorname{argmax}\left\{(\tau_{iu})^{\alpha}.(\eta_{iu})^{\beta}.(\gamma_{iu})^{\lambda}\right\} & if\ q \le q_0 \\ J\ if\ q \succ q_0 \end{cases} \quad (1)$$

J is a random variable generated according to the probability distribution function given by:

$$p_{ij} = \begin{cases} \dfrac{(\tau_{ij})^{\alpha} \cdot (\eta_{ij})^{\beta} \cdot \ (\gamma_{ij})^{\lambda}}{\sum\limits_{u \neq F_k} (\tau_{iu})^{\alpha} \cdot (\eta_{iu})^{\beta} \cdot \ (\gamma_{iu})^{\lambda}} & \text{if } u \in F_k \\ 0 & \text{otherwise} \end{cases} \quad (2)$$

where q is a random number uniformly distributed in [0, ..., 1] and q_0 is a parameter $(0 \le q_0 \le 1)$. τ_{ij} is the pheromone associated with arc (i, j), η_{ij} is the heuristic desirability, known as visibility, and is the local heuristic function which is the inverse of the distance between customer i and j. The selection probability is then further extended by problem specific information. There, the inclusion of savings leads to better results. The saving function [11] is represented by $\gamma_{ij} = d_{i0} + d_{0j} - g.d_{ij} + f.\left|d_{i0} - d_{0j}\right|$ where f and g are two parameters. F_k is the set of feasible customers that remain to be visited by ant k and α, β and λ are a parameters which determine the relative importance of the trails, distance and the savings heuristic, respectively. The parameter q_0 determines the relative importance of exploitation against exploration. Before an ant selects the next customer to visit the random number q is generated. If $q \le q_0$ then exploitation is encouraged, whereas $q > q_0$ encourages biased exploration.

3.3.2. Update pheromone trail. While an ant is building its solution, the pheromone level on each arc (i, j) that is visited is updated according to the local updating rule given in Equation (3).

$$\tau_{ij} = (1 - \rho) + \rho.\Delta\tau_{ij} \quad (3)$$

Where ρ is a parameter $(0 < \rho < 1)$ and $\Delta\tau_{ij} = \tau_0$ the initial pheromone trail.

Once all ants have built their tours then the global updating rule is applied. In the ACS method only the globally best ant is allowed to deposit pheromone in an attempt to guide the search. The global updating rule is given in the Equation (4).

$$\tau_{ij}^{new} = (1-\rho)\tau_{ij}^{old} + \rho\Delta\tau_{ij} \quad (4)$$

Where $0 < \rho < 0$ is the pheromone decay parameter, If arc (i, j) is used by the best ant then the pheromone trail is increased on that arc by $\Delta\tau_{ij}$ which is equal to $1/L^*$, where L^* is the length of the tour produced by the best ant

After the ant system is initialised, the steps detailed above are repeated for a given number of iterations. Once each ant has produced a set of routes the local optimizer 2-opt is applied to improve the solutions if possible.

(1) Initialization

- Open randomly one depot ;
- The initial solution s_0 **;**
- $s \leftarrow s_0$;
- $TCost \leftarrow +\infty$; $BestCost \leftarrow TCost$;
- $TabuList \leftarrow \Phi$;

(2) Improvement

Repeat

Location phase

- apply the swap and add moves to calculate the neighbourhood ($N(s)$) of the current solution s ;
- choose $s' \in \arg\min\{\cos t(N(s))\}$ with $s' \notin TabuList$;
- if $Cost(S') < Cost(S)$ then
  - $s \leftarrow s'$;

Routing phase:

  - $Y' \leftarrow ACS(S)$ (calculate the route of vehicles));
  - $TCost' \leftarrow Cost(Y') + FixedCost(opened\ depots)$;
  - if ($TCost' < TCost$) then
    - $BestSol \leftarrow Y'$;
    - $BestCost \leftarrow TCost'$;
  - end if;
  - update the $TabuList$
- end if;

Until stop condition

Return $BestSol$

Figure 2 Pseudo-code of the Tabu search and Ant Colony system algorithm proposed to the CLRP

4. Computational results

To evaluate the Tabu-Ant algorithm, computational tests were carried out on 18 CLRP instances (see Table 1) obtained from the literature (Or, 1976; Perl, 1983) or adapted from data related to VRP (Gaskell, 1967; Christofides and Eilon, 1969; Min et al., 1992; Daskin, 1995). Data relative to the used instances are available in Barreto (2004).

Table 1 gives the computational results for the test problems obtained by our approach compared to the best solution published, where the CLRP instance column contains information about the author, the publication year and the author and the number of customers and potential Distribution centers. The best published cost column contains the best result obtained running a set of versions of a clustering "routing first-location second" heuristic in Barreto et al. (2007). The Tabu-Ant cost column contains the best result obtained running our approach. The last column represents the deviation, where

$$deviation = \frac{Solution(Tabu\_Ant) - Solution(Best\ published)}{Solution(Best\ published)} * 100$$

Table 1 Computational Results

CLRP Instance	Best published	Tabu_Ant	Deviation
Gaskell67-21x5	435.9	**429.72**	-1,42%
Gaskell67-22x5	591.5	**585.10**	-1,15%
Gaskell67-29x5	**512.1**	**512.1**	0%
Gaskell67-32x5	571.7	**566.19**	-0,96%
Gaskell67-32x5_2	511.4	**506.13**	-1,03%
Gaskell67-36x5	470.7	**470.42**	-0,06%
Min92-27x5	**3062**	**3062**	0%
Min92-134x8	6238.0	**6208.78**	-0,47%
Perl83-12x2	**204**	**204**	0 %
Perl83-55x15	1136.2	**1118.43**	-1,56%
Perl83-85x7	1656.9	**1647.05**	-0,34%
Perl83-Cli-318x4	**747619**	748238.67	0,08 %
Christofides69-50x5	582.7	**577.56**	-0,88 %
Christofides69-75x10	886.3	**880.94**	-0,6%
Christofides69-100x10	**889.4**	895.61	0,7 %
Daskin95-88x8	**384.9**	**384.9**	0%
Daskin95-150x10	**46642.7**	46705.27	0 ,14%
Or76-117x14	**12474.2**	12589.14	0,92 %

From the table 1, the results obtained by our approach improve, in the most of the cases, the best results published obtained by clustering "routing first-location second" heuristic in Barreto et al. (2007). The average deviation of the solutions obtained by our algorithm is (-0.35%) compared to the best solution known on the whole of the tests. Hence, our approach is efficient and competitive.

For the setting parameters we use 10 artificial ants, $\alpha = 1, \beta = 2, \lambda = 1$, $\rho = 0.1$, $q_0 = 0.75$ and $f = g = 2$ (for the savings function). The algorithm have been coded on java and executed on a Pc portable, with Windows XP system, processor P4 of 2.4 GHz and 512 Mo of RAM.

5. Conclusion

In this paper, we have presented a new algorithm for the CLRP. The algorithm combines two meta-heuristics, a Tabu search and a hybrid Ant colony system which cooperating together to optimize the cost of location and routing. The tabu search (TS) is used to find a good configuration of distribution centers (DC), and the ant colony system (ACS) is used to find the good routing corresponding to this configuration

Preliminary results show that our algorithm outperforms other algorithms by producing the best results for the set of tested instances. The paper shows the successful application of the tabu search and the ant colony system to the capacitated location routing problem.

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